MA 3232 - Numerical Analysis
Sample Exam I

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. One page ($8\frac{1}{2}$ by 11) of notes (both sides) permitted.

---

1. (25 points) Consider the function:

$$f(x) = 5x - e^{-x}$$

a. Why must at least one root of this function exist in the interval $0 < x < 1$?

b. Estimate the value of this root using two iterations of Newton's method and a starting value $x_0 = 0$.

c. Estimate the error in your answer to part b.

d. Would the method of linear iteration have been appropriate for this problem? If so, would it have been preferable to Newton's method? (Briefly *explain* your answer.)

---

2. (25 points) Consider the following table of data:

x_i	f_i	Δf_i
0.0000	0.0000	0.6065
0.2500	0.6065	0.1293
0.5000	0.7358	–0.0664
0.7500	0.6694	–0.1281
1.0000	0.5413	–0.1309
1.2500	0.4104	–0.1117
1.5000	0.2987	

a. Using the most appropriate second degree Newton-Gregory interpolating polynomial, approximate $f(0.81)$.

b. Estimate the error in your answer to part a.

c. If you had used a second degree Lagrange polynomial on the same data points, would your answer to part a. have changed? (Briefly *explain* your answer!)

---

3. (25 points) a. Consider the difference approximation

$$f'(x_n) \doteq \frac{f_{n+1} - f_{n-1}}{2h}$$

(1) Use this formula to approximate the derivative of $f(x) = \sin(x)$ at $x = 0$ using step sizes of $h = 0.10$ and 0.20.

(2) Is the error in your answers consistent with the expected order of this method. (*Briefly* explain your answer)

b. In general, difference approximations to the first derivative are produced according to the formula

$$f'(x_i) \doteq \frac{1}{h} \left. \frac{dP_n}{ds} \right|_{s=i}$$

If this formula were used to derive a difference approximation for

$$f'(x_3)$$

based on values at x_0, x_1, x_2, and x_3, what order would you expect it to be? Would it be forward, backward, or central? (You do **not** have to derive the difference approximation to answer this question!)

---

4. (25 points) a. Compute

$$2.95x^2 - 1.13x + 2.25$$

for $x = 1.07$, as it would be computed by a four digit, decimal computer which *chops* all computations (including *intermediate* values).

b. Many root-finding algorithms commonly stop when the current iterate (x_n) produces an acceptably small residual, i.e. when

$$f(x_n) \quad \text{is "small."}$$

Is the effectively a forward or a backward error test? (*Briefly* justify your answer.)

c. (1) For what values of x will catastrophic cancellation be a potential problem in the expression:

$$\sqrt{x^4 - 1} - (x - 1)^2$$

(2) Can you find a numerically "better" equivalent form for this expression?
